

A business wants to manufacture bike frames. Before they start production, they need to make sure they can make a profit with the materials and labor force they have. Their accountant has given them a cost equation of

$$y = 7,000 + 0.85x + 30,000$$

and a revenue equation of

$$y = 2x - 0.45x + 2000$$

Determine the break-even point for the company.

$$\begin{aligned}
 7000 + 0.85x + 30000 &= 2x - 0.45x + 2000 \\
 37,000 + 0.85x &= 1.55x + 2000 \\
 -0.85x &\quad -0.85x \\
 \hline
 37000 &= 0.7x + 2000 \\
 -2000 &\quad -2000 \\
 \hline
 35000 &= 0.7x \\
 \frac{35000}{0.7} &= \frac{0.7x}{0.7} \\
 50,000 &= x
 \end{aligned}$$

THE COMPANY NEEDS TO SELL 50,000 BIKE FRAMES TO BREAK EVEN.

Identifying 1 solution, no solutions, or infinitely many solutions for equations with variables on both sides by inspection:

When you combine LIKE TERMS on each side of the equation, and then move the VARIABLE to one side of the equation, one of two things will happen:

Either the VARIABLES will CANCEL or they won't.

If they don't CANCEL, the equation has 1 solution.

If they do CANCEL, are you left with a true statement or a false statement?

If you're left with a TRUE statement, the equation has infinitely many solutions. $4000 = 4000$

If you're left with a FALSE statement, the equation has no solutions. $10000 = 0$

↑
FALSE

TRUE
↓

Create an equation that has no solutions.

(show it has no solutions by solving it and being left with a false statement)

$$\begin{array}{r} \cancel{5x} + 100 = 5x \\ -\cancel{5x} \quad \quad \quad -\cancel{5x} \\ \hline 100 = 0 \leftarrow \text{FALSE} \end{array}$$

NO SOLUTIONS

Create an equation that has infinitely many solutions.

(show it has infinitely many solutions by solving it and being left with a true statement)

$$\begin{array}{r} 1 + 2x + 3 = x + 4 + x \\ \cancel{2x} + 4 = \cancel{2x} + 4 \\ -\cancel{2x} \quad \quad \quad -\cancel{2x} \\ \hline 4 = 4 \leftarrow \text{TRUE} \end{array}$$

INFINITELY
MANY
SOLUTIONS