3-3 Parallel and Perpendicular Lines

What You'll Learn

- To relate parallel and perpendicular lines

. . . And Why

To show how to fit sides of a picture frame together, as in Example 1

Check Skills You'll Need

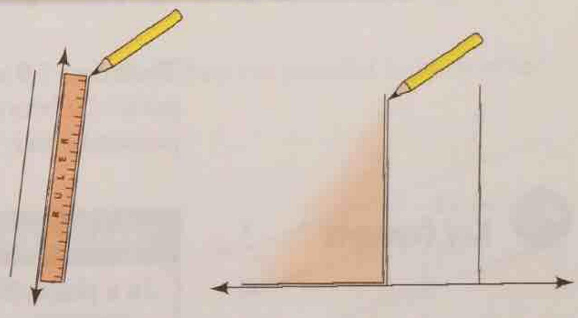
GO for Help Lesson 1-4

Complete each statement with *always, sometimes,* or *never.*

1. Two lines in the same plane are _?_ parallel.
2. Perpendicular lines _?_ meet at right angles.
3. Two lines in intersecting planes are _?_ perpendicular.
4. Two lines in parallel planes are _?_ perpendicular.

1 Relating Parallel and Perpendicular Lines

The two diagrams suggest ways to draw parallel lines. You can draw them (a) parallel to a given line, or (b) perpendicular to a given line. Theorems 3-9 and 3-10 guarantee that the lines you draw are indeed parallel.

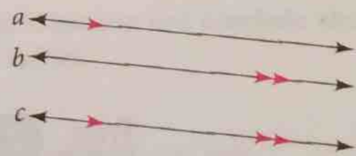


Key Concepts

Theorem 3-9

If two lines are parallel to the same line, then they are parallel to each other.

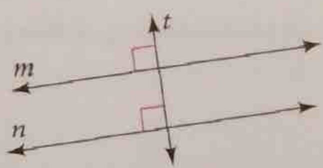
$a \parallel b$



Theorem 3-10

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

$m \parallel n$

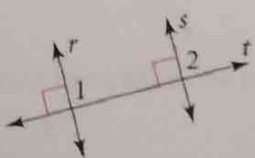


Theorem 3-10 includes the phrase *in a plane*. On the other hand, Theorem 3-9 is true for any three such lines, whether they are coplanar (Exercise 3) or noncoplanar.

Proof

Proof of Theorem 3-10

Study what is given, what you are to prove, and the diagram. Then write a paragraph proof.



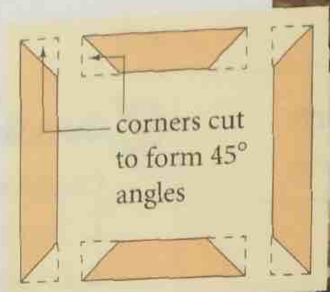
Given: $r \perp t, s \perp t$

Prove: $r \parallel s$

Proof: $\angle 1$ and $\angle 2$ are right angles by the definition of perpendicular, so they are congruent. Since corresponding angles are congruent, $r \parallel s$.

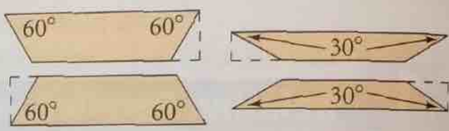
1 EXAMPLE Real-World Connection

Woodworking To make a frame for a painting, a miter box and a backsaw are used to cut the framing at 45° angles. Explain why cutting the framing at this angle ensures that opposite sides of the frame will be parallel.



Two adjacent 45° angles form a 90° angle. The opposite sides of the frame are perpendicular to the same side. Thus the opposite sides are parallel because two lines perpendicular to a third line are parallel.

Quick Check 1 Can you assemble the framing at the right into a frame with opposite sides parallel? Explain.



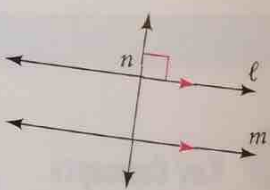
Theorems 3-9 and 3-10 gave conditions by which you can conclude that lines are parallel. Theorem 3-11 provides a way for you to conclude that lines are perpendicular. You will prove Theorem 3-11 in Exercise 11.

Key Concepts

Theorem 3-11

In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.

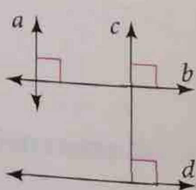
$n \perp m$



Proof

2 EXAMPLE Using Theorem 3-11

Study what is given, what you are to prove, and the diagram. Then write a paragraph proof.



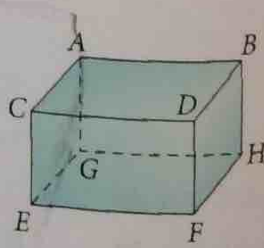
Given: In a plane, $a \perp b$, $b \perp c$, and $c \perp d$.

Prove: $a \perp d$

Proof: Lines a and c are both perpendicular to line b, so $a \parallel c$ because two lines perpendicular to the same line are parallel. It is given that $c \perp d$. Therefore, $a \perp d$ because a line that is perpendicular to one of two parallel lines is perpendicular to the other (Theorem 3-11).

Quick Check 2 From what is given in Example 2, can you also conclude $b \parallel d$? Explain.

In the rectangular solid shown here, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are parallel. $\overleftrightarrow{EC}$ is perpendicular to $\overleftrightarrow{AC}$, but it is not perpendicular to $\overleftrightarrow{BD}$. This is why Theorem 3-11 states that the lines must be "in a plane."



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

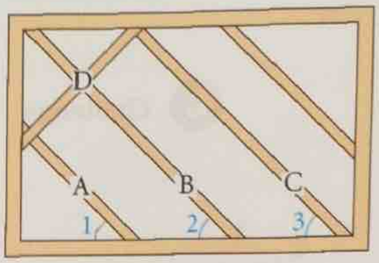
Practice and Problem Solving

A Practice by Example

Example 1 (page 142)

GO for Help

1. A carpenter is building a trellis for vines to grow on. The completed trellis will have two sets of overlapping diagonal slats of wood.

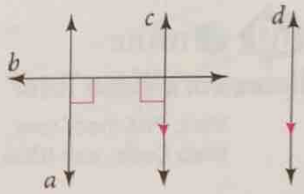


a. What must be true of $\angle 1$, $\angle 2$, and $\angle 3$ if slats A, B, and C must be parallel?

b. The carpenter attaches slat D so that it is perpendicular to slat A. Is slat D perpendicular to slats B and C? Justify your answer.

Example 2 (page 142)

Proof **2.** Study what is given, what you are to prove, and the diagram. Then write a proof.



Given: In a plane, $a \perp b, b \perp c$, and $c \parallel d$.

Prove: $a \parallel d$

B Apply Your Skills

GO for Help

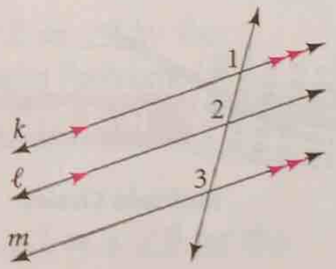
For a guide to solving Exercise 3, see p. 145.

3. Developing Proof Copy and complete this paragraph proof of Theorem 3-9 for three coplanar lines.

If two lines are parallel to the same line, then they are parallel to each other.

Given: $\ell \parallel k$ and $m \parallel k$

Prove: $\ell \parallel m$



Proof: $\ell \parallel k$ means that $\angle 2 \cong \angle 1$ by the **a.** __?__ Postulate. $m \parallel k$ means that **b.** __?__ $\cong$ **c.** __?__ for the same reason. By the Transitive Property of Congruence, $\angle 2 \cong \angle 3$. By the **d.** __?__ Postulate, $\ell \parallel m$.

Each of the following statements describes a ladder. What can you conclude about the rungs, one side, or both sides of each ladder? Explain.

Real-World Connection

This ladder's rungs are perpendicular to each side. Therefore, the rungs are parallel to each other.

4. The rungs are each perpendicular to one side.

5. The rungs are parallel and the top rung is perpendicular to one side.

6. The sides are parallel. The rungs are perpendicular to one side.

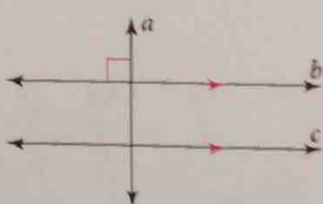
7. The rungs are perpendicular to one side. The other side is perpendicular to the top rung.

8. Each side is perpendicular to the top rung.

9. Each rung is parallel to the top rung.

10. The rungs are perpendicular to one side. The sides are not parallel.

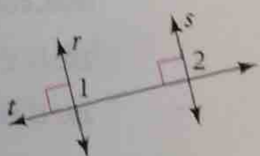
Proof **11.** Prove Theorem 3-11: In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.



Given: In a plane, $a \perp b$, and $b \parallel c$.

Prove: $a \perp c$

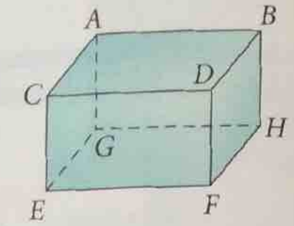
Proof **12.** Prove: If a line is perpendicular to each of two other lines, all in one plane, then the two other lines are parallel.



Given: $t \perp r, t \perp s$

Prove: $r \parallel s$

13. **Writing** Theorem 3-10: In a plane, two lines perpendicular to the same line are parallel. Use the rectangular solid at the right to explain why the words *in a plane* are needed.



C Challenge

a, b, c, and d are distinct lines in the same plane. Exercises 14-21 show different combinations of relationships between a and b, b and c, and c and d. For each combination of the three relationships, how are a and d related?

14. $a \parallel b, b \parallel c, c \parallel d$
15. $a \parallel b, b \parallel c, c \perp d$
16. $a \parallel b, b \perp c, c \parallel d$
17. $a \perp b, b \parallel c, c \parallel d$
18. $a \parallel b, b \perp c, c \perp d$
19. $a \perp b, b \parallel c, c \perp d$
20. $a \perp b, b \perp c, c \parallel d$
21. $a \perp b, b \perp c, c \perp d$

GO Online
Homework Video Tutor
Visit: PHSchool.com
Web Code: aue-0303

Critical Thinking **The Reflexive, Symmetric, and Transitive Properties for Congruence (≅) are listed on page 105.**

22. Write reflexive, symmetric, and transitive statements for "is parallel to" ($\parallel$). State whether each statement is true or false and justify your answer.

23. Repeat Exercise 22 for "is perpendicular to" ($\perp$).

Test Prep

Multiple Choice

24. In a plane, line e is parallel to line f, line f is parallel to line g, and line h is perpendicular to line e. Which of the following MUST be true?
    A. $e \parallel g$ B. $h \parallel f$ C. $g \parallel h$ D. $e \parallel h$

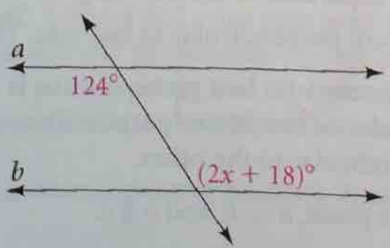
25. In a plane, $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$. $\angle ABC$ is a right angle. What can you conclude?
    I. $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ II. $\overleftrightarrow{BC} \perp \overleftrightarrow{CD}$ III. $\overleftrightarrow{BC} \perp \overleftrightarrow{AD}$
    F. I only G. I and II H. II only J. I, II and III

Mixed Review

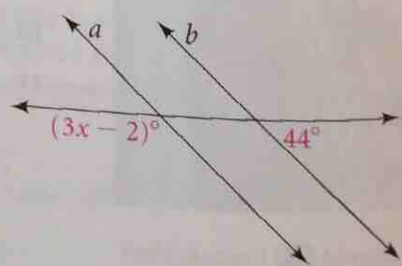
GO for Help

Lesson 3-2 Algebra **Determine the value of x for which $a \parallel b$.**

26.



27.



Lesson 2-2 **Each conditional statement below is true. Write its converse. If the converse is also true, combine the statements into a biconditional.**

28. If $x = 7$, then $x^2 = 49$.

29. If two lines in a plane do not meet, then the lines are parallel.