

3.2 Angles of Triangles

Learning Target: Understand properties of interior and exterior angles of triangles.

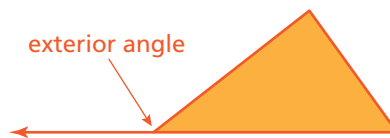
- Success Criteria:**
- I can use equations to find missing angle measures of triangles.
 - I can use interior and exterior angles of a triangle to solve real-life problems.

EXPLORATION 1

Exploring Interior and Exterior Angles of Triangles

Work with a partner.

- Draw several triangles using geometry software. What can you conclude about the sums of the angle measures?
- You can extend one side of a triangle to form an *exterior angle*, as shown.



Use geometry software to draw a triangle and an exterior angle. Compare the measure of the exterior angle with the measures of the interior angles. Repeat this process for several different triangles. What can you conclude?

EXPLORATION 2

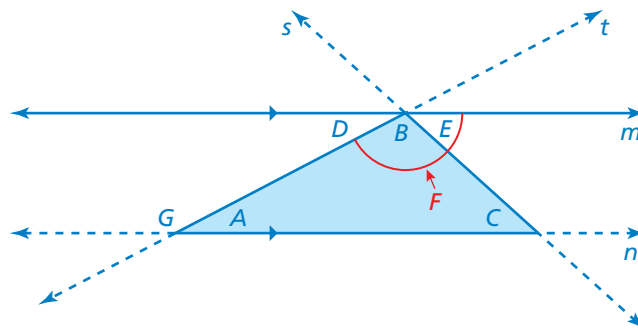
Using Parallel Lines and Transversals

Work with a partner. Describe what is shown in the figure below. Then use what you know about parallel lines and transversals to justify your conclusions in Exploration 1.

Math Practice

Look for Structure

Which angle labeled in the diagram is an exterior angle of $\triangle ABC$?



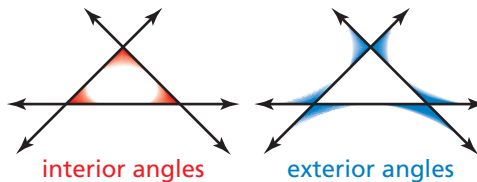
3.2 Lesson

Key Vocabulary

interior angles of a polygon, p. 112

exterior angles of a polygon, p. 112

The angles inside a polygon are called **interior angles**. When the sides of a polygon are extended, other angles are formed. The angles outside the polygon that are adjacent to the interior angles are called **exterior angles**.

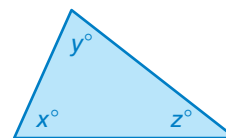


Key Idea

Interior Angle Measures of a Triangle

Words The sum of the interior angle measures of a triangle is 180° .

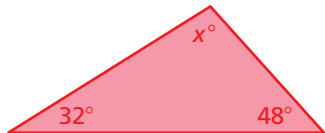
Algebra $x + y + z = 180$



EXAMPLE 1 Using Interior Angle Measures

Find the measures of the interior angles of each triangle.

a.



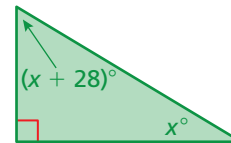
$$x + 32 + 48 = 180$$

$$x + 80 = 180$$

$$x = 100$$

▶ So, the measures of the interior angles are 100° , 48° , and 32° .

b.



$$x + (x + 28) + 90 = 180$$

$$2x + 118 = 180$$

$$2x = 62$$

$$x = 31$$

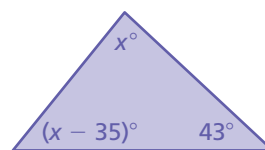
▶ So, the measures of the interior angles are $(31 + 28)^\circ = 59^\circ$, 31° , and 90° .

Try It Find the measures of the interior angles of the triangle.

1.



2.

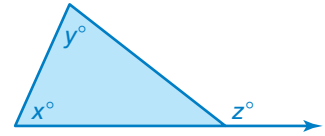


Key Idea

Exterior Angle Measures of a Triangle

Words The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

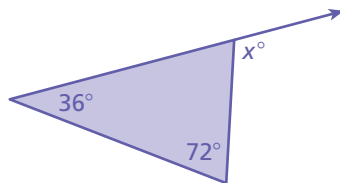
Algebra $z = x + y$



EXAMPLE 2 Finding Exterior Angle Measures

Find the measure of the exterior angle.

a.



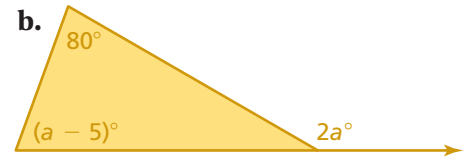
$$x = 36 + 72$$

$$x = 108$$



So, the measure of the exterior angle is 108° .

b.



$$2a = (a - 5) + 80$$

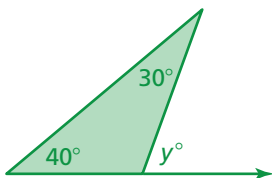
$$2a = a + 75$$

$$a = 75$$



So, the measure of the exterior angle is $2(75)^\circ = 150^\circ$.

Each vertex has a pair of congruent exterior angles. However, it is common to show only one exterior angle at each vertex.



Try It

3. Find the measure of the exterior angle of the triangle at the left.



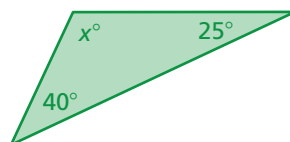
Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

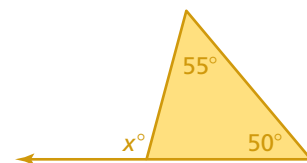
4. **VOCABULARY** How many exterior angles does a triangle have at each vertex? Explain.

FINDING ANGLE MEASURES Find the value of x .

5.



6.



EXAMPLE 3

Modeling Real Life

An airplane leaves Miami and travels around the Bermuda Triangle as shown in the diagram. What is the measure of the interior angle at Miami?

Understand the problem.

You are given expressions representing the interior angle measures of the Bermuda Triangle. You are asked to find the measure of the interior angle at Miami.

Make a plan.

Use what you know about interior angle measures of triangles to write and solve an equation for x .

Solve and check.

$$x + (2x - 44.8) + 62.8 = 180$$

$$3x + 18 = 180$$

$$3x = 162$$

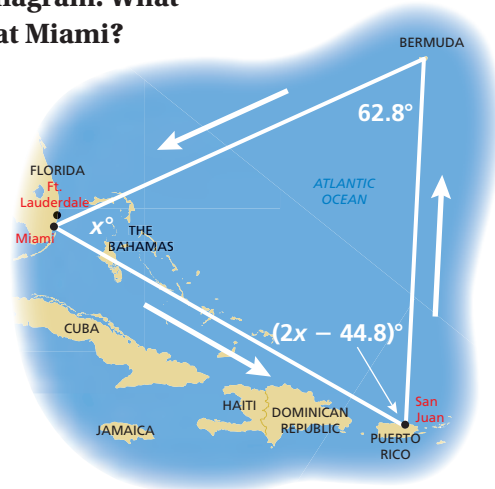
$$x = 54$$

Write an equation.

Combine like terms.

Subtract 18 from each side.

Divide each side by 3.



Check

$$x + (2x - 44.8) + 62.8 = 180$$

$$54 + [2(54) - 44.8] + 62.8 \stackrel{?}{=} 180$$

$$54 + 63.2 + 62.8 \stackrel{?}{=} 180$$

$$180 = 180 \checkmark$$

So, the measure of the interior angle at Miami is 54° .

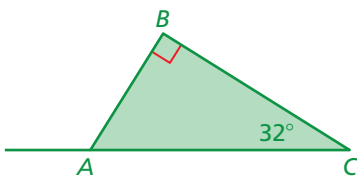


Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. The *Historic Triangle* in Virginia connects Jamestown, Williamsburg, and Yorktown. The interior angle at Williamsburg is 120° . The interior angle at Jamestown is twice the measure of the interior angle at Yorktown. Find the measures of the interior angles at Jamestown and Yorktown. Explain your reasoning.

8. A helicopter travels from point C to point A to perform a medical supply drop. The helicopter then needs to land at point B . How many degrees should the helicopter turn at point A to travel towards point B ? Justify your answer.



3.2 Practice

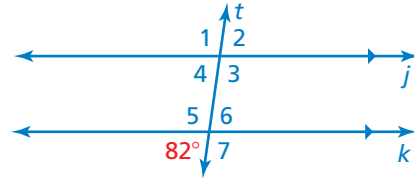


Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

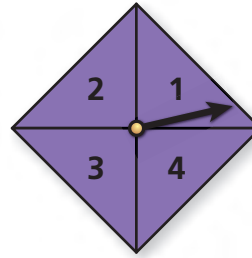
Use the figure to find the measure of the angle.
Explain your reasoning.

1. $\angle 2$
2. $\angle 6$
3. $\angle 4$
4. $\angle 1$



You spin the spinner shown.

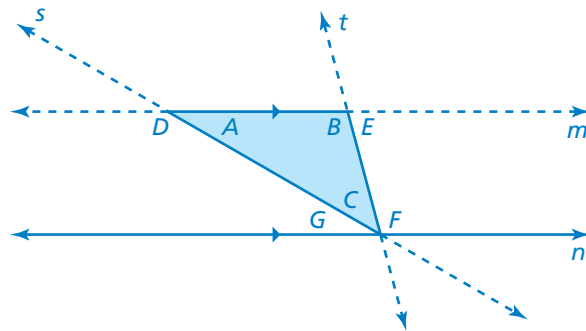
5. What are the favorable outcomes of spinning a number less than 4?
6. In how many ways can spinning an odd number occur?



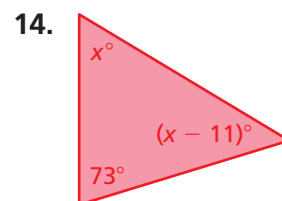
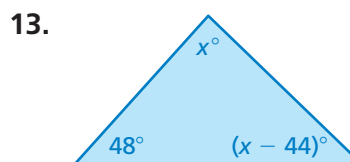
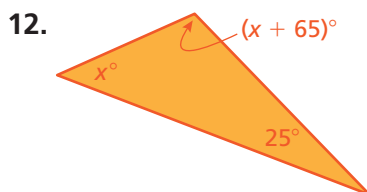
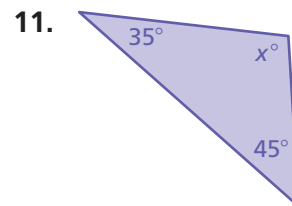
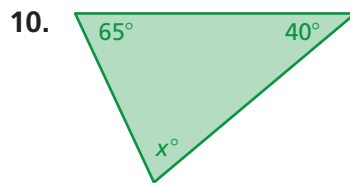
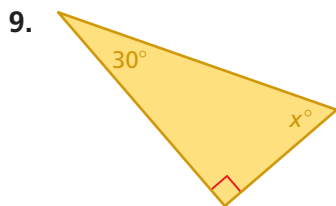
► Concepts, Skills, & Problem Solving

USING PARALLEL LINES AND TRANSVERSALS Consider the figure below.
(See Exploration 2, p. 111.)

7. Use a protractor to find the measures of the labeled angles.
8. Is $\angle F$ an exterior angle of Triangle ABC ? Justify your answer.

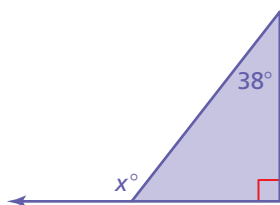


USING INTERIOR ANGLE MEASURES Find the measures of the interior angles of the triangle.

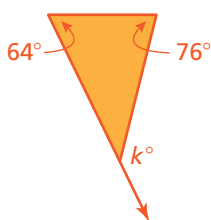


FINDING EXTERIOR ANGLE MEASURES Find the measure of the exterior angle.

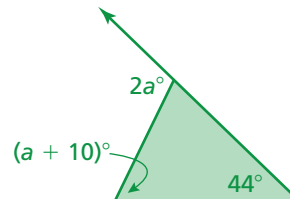
15.



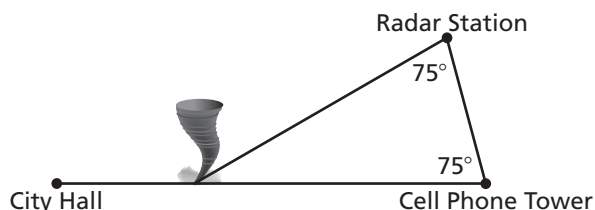
16.



17.



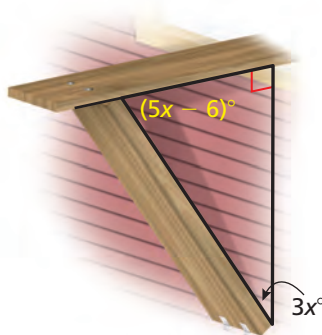
18. **MODELING REAL LIFE** A tornado is located between city hall and a cell phone tower and is heading towards the cell phone tower. By what angle does the tornado's direction need to change so that it passes over the radar station instead? Justify your answer.



19. **YOU BE THE TEACHER** Your friend finds the measure of the exterior angle shown. Is your friend correct? Explain your reasoning.

$(3x - 6) + x + 30 = 180$
 $4x + 24 = 180$
 $x = 39$

The exterior angle is $(3(39) - 6)^\circ = 111^\circ$.



20. **MP REASONING** The ratio of the interior angle measures of a triangle is 2 : 3 : 5. What are the angle measures?
21. **MP PROBLEM SOLVING** The support for a window air-conditioning unit forms a triangle and an exterior angle. What is the measure of the exterior angle?
22. **MP REASONING** A triangle has an exterior angle with a measure of 120° . Can you determine the measures of the interior angles? Explain.

ANGLES OF TRIANGLES Determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

23. Given three angle measures, you can construct a triangle.
24. The acute interior angles of a right triangle are complementary.
25. A triangle has more than one vertex with an acute exterior angle.

26. **DIG DEEPER!** Using the figure at the right, show that $z = x + y$. (Hint: Find two equations involving w .)

